

Покажите, что последовательность $b_n = (1 - 1/n)^n$ возрастающая

$$(1+x)^k \geq 1+kx$$

$$b(n) = (1 - 1/n)^n$$

$$b(n+1) = (1 - 1/(n+1))^{n+1}$$

$$b(n+1) / b(n) = (1 - 1/(n+1))^{n+1} / (1 - 1/n)^n = (n/(n+1))^{n+1} / ((n-1)/n)^n =$$

$$n^{2n+1} / ((n+1)^{n+1} * (n-1)^n) = n^{2n+1} / ((n+1)^n * (n-1)^n * (n+1)) = n^{2n+1} / ([n(n-1)]^n * (n+1)) = n^{2n+1} / ([n^2-1]^n * (n+1)) =$$

$$= n/(n+1) * n^{2n} / [n^2-1]^n = n/(n+1) * [n^2/(n^2-1)]^n = n/(n+1) * [(n^2-1+1)/(n^2-1)]^n =$$

$$= n/(n+1) * [1 + 1/(n^2-1)]^n = \text{применяем неравенство бернулли} \Rightarrow n/(n+1) * (1 + n/(n^2-1)) \geq$$

$$\geq n/(n+1) * (1 + n/n^2) = n/(n+1) * (1 + 1/n) = n/(n+1) * (n+1)/n = 1$$